The Market for Credit Ratings: Competition and Misalignment of Interests

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Abstract
We examine the theoretical effects of competition on credit rating agencies. Contrary to prior literature, we make the realistic assumption that rated firms’ excess profits flow to their shareholders. This change makes credit rating agencies compete more fiercely for market share when building a reputation for rating accuracy. Our model permits both a closed-form three-period solution, and a numerically simulated infinite-horizon solution. We reaffirm the finding that the disciplining effect of reputation is insufficient to ensure truthful reporting by credit rating agencies. Though competition in most instances aggravates lax behaviour by credit rating agencies, the fiercer competition reduces this effect.

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1 Introduction

A credit rating agency’s (CRA) main task is to provide a symbol that, regardless of the type of security, consistently indicates the expected credit loss. The CRA works as an information intermediary by transferring private information from the debt issuing firm to the investor in a credible manner. Their ratings provide a quick way, through a well-known scale, for uninformed investors to assess the credit risk across many financial assets. Use of ratings avoids costly duplication efforts by investors. Truthful CRAs therefore fulfill an important task in the allocation of capital in an efficient and cost-effective way on the world financial markets.

The business model of a CRA changed fundamentally in the 1970s from an investor-pay model to an issuer-pay model. The introduction of the photocopy-machines allowed investors to cheaply copy ratings books thus deteriorating the CRA’s income. This fundamental change to the business model shifted the CRAs alignment of incentives towards the issuers away from the investors.

In the wake of the financial crisis attention has been diverted towards the credit rating industry for their failure to predict the collapse of highly rated assets and corporations. The issuer-pay model and its misalignment of incentives between the debt issuing firm, the credit rating agency, and the investor became a focal point.

To reduce these potential incentive problems, increased competition amongst credit rating agencies has been called for. Especially in the European Union policy makers have been vocal about this remedy.

However an empirical study by Becker and Milbourn (2011) indicates increased competition tends to cause CRA’s to inflate ratings, that is they assign assets better ratings, than they objectively merit. In this article we study the theoretical background for Becker and Milbourn (2011) findings.

We develop a reputation model well-known from the industrial organisation literature. Repeated interaction between firms and consumers allow firms to build up a reputation for selling a high quality good.

However, producing quality goods are for various reasons costly. A trade-off arises between long-run reputation and shirking on quality in the short run. In the context of credit ratings, a feature of the market is; the issuing firm can deny the CRA its rating fee by choosing not to publish an unfavourable rating report. A CRA must balance its long-run reputation of delivering correct ratings and collecting income from inflating ratings in the short-run. According to S&P
The ongoing value of Standard & Poor’s credit ratings business is wholly dependent on continued market confidence in the credibility and reliability of its credit ratings. No single issuer fee or group of fees is important enough to risk jeopardizing the agency’s reputation and its future.

The academic literature points to misaligned incentives between the participants in a complex market with many conflicting interests. The literature indicates that the solution to the trade-off is not as clear cut as proposed by S&P. The complexity has resulted in a, so far somewhat limited, stylised literature with a narrow focus in each article.

Mathis et al. (2009) seminal study on the credit ratings market develops a theoretical closed model for a monopoly CRA. They show reputation is an insufficient disciplining mechanism to ensure truthful rating reports. Unless a large fraction of the CRA’s income stems from other sources than rating complex securities, the CRA will tend to inflate ratings. Reputation will follow a cyclical movement, i.e. in economic booms a CRA will tend to increase lax behavior. Further, if the CRAs observation technology is imperfect, it will always lie with positive probability.

Becker and Milbourn (2011) empirical study of the competition on the market for credit ratings find increased competition decreased the quality of the ratings. Exploiting the natural experiment of Fitch’s emergence as a major credit rating agency, they find that rating levels went up over the period from 2000–07, while the correlation between rating levels and market-implied yield fell and the ability of ratings to predict default worsen. Their findings are made stronger by ruling out rating shopping.

This exact empirical finding is what this article aim to study in a theoretical framework.

Camanho et al. (2011) study of the effects of competition on the quality of the ratings lays a theoretical background for Becker and Milbourn (2011)’s findings. Their model exhibits the property of Mathis et al. (2009) where long run reputation is not a sufficient disciplining mechanism to ensure truthful rating reports by rational credit rating agencies. Camanho et al. (2011) new finding shows increased competition weakens the disciplining effect of reputation compared to a monopoly situation. Further they show expected welfare is decreased by competition. However their findings come at the expense of a closed form solution.

Camanho et al. (2011) assume that investors extract all rent, which we find unlikely to be

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1S&P - U.S. Securities and Exchange Commission Public Hearing - November 15, 2002
the case in the actual market. We make the more realistic assumption, that rated firms’ excess profits flow to their shareholders.

Our review of the recent literature shows the conflicting interest of the market participants. There are two main underlying assumptions. One focuses on the conflict of interest arising from the issuer pay model causing CRAs to inflate ratings to attract business. The CRAs themselves claim reputational concerns trump any one short-term gain from lying. Thus this part of the literature examines reputation as a disciplining mechanism. Building on the seminal papers by Shapiro (1983) and Hörner (2002), the mainstream view of this part of literature suggests that a rational CRA does not have strong enough reputational concerns to abstain them from lying.

This view is present in the work by Doh-Shin and Lovo (2011) and Bolton et al. (2012). The later considers the effect of competition on the market for credit ratings when investors can shop for ratings. They find competition increases rating inflation. However this property can be alleviated by a mandatory ratings disclosure such as the Cuomo-plan\(^2\). Doh-Shin and Lovo (2011), being an exception, show the threat of entry can sufficiently strengthen the reputational concerns.

The other part of the literature assumes that CRAs genuinely are honest, striving to be as accurate as possible. However failure to always predict the right outcome may stem from multiple reasons. Skreta and Veldkamp (2010) show ratings inflation can be caused by the issuing firms. The issuing firm takes advantage of competing truthful but imprecise CRAs. As imprecision increases with asset complexity, issuers will choose to increase asset complexity to enhance the possibility that one of the CRAs provide a too good rating. Manso (2011) points to economic cycles as cause for inaccurate ratings, creating a scope for multiple accurate ratings. The interest rate paid depends on the rating. Thus providing a good rating may be accurate as long as the implied interest payment is low enough, but a small shock to the economy may cause the firm to be unable to service the debt, which causes the CRA to downgrade the firm to a new, also accurate, rating.

Finally Bar-Isaac and Shapiro (2011a) and Bar-Isaac and Shapiro (2011b) advocate that CRAs’ inability to deliver accurate ratings can be due to labour market constraints. The competition for high quality workers between CRAs and investment banks results in worsen rating accuracy, when good analysts are lured into investment banks by high wages.

\(^2\)Doh-Shin and Lovo (2011) p. 2: “The Cuomo plan, which is an agreement between New York State Attorney General Andrew Cuomo and the three main CRAs, requires that the issuers pay CRAs upfront for their rating, not contingent on the report”.

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The paper is organised as follows: In Section 2, we propose our model, and analyse the finite horizon solution for both the monopoly and duopoly case in section 3. In Section 4 we simulate a dynamic numerical solution for both cases and Section 5 concludes.

2 Model

We examine reputational concerns as a disciplining mechanism, when competition is introduced to the credit rating market, inspired by the articles by Mathis et al. (2009) and Camanho et al. (2011).

More formally we adopt a similar framework as Camanho et al. (2011), but make a fundamental change. Namely we let the issuing firm owner be the residual recipient of the profits stemming from a successful project. Investors are instead assumed to be risk neutral competing on perfect capital markets, where interest rates are determined by zero profit conditions.

To create a scope for competition we adopt a differentiated products Bertrand approach, known from the Industrial Organisation literature. This can be motivated by either the notion of locked-in relationship between issuer and rating agency through real or perceived switching costs, segmented markets or subjective irrational preference for one agency over another.

2.1 Market characteristics

In the model we consider three types of agents: issuers of debt, credit rating agencies (CRA) and investors. It runs for a number of discrete periods, during each period \( t = 0, 1, 2, \ldots \).

A cashless firm seeks to issue debt to finance a project. The project costs \( c \sim u(c_L, c_H) \) to finance. The quality of the project is unknown a priori to all agents in the economy. The quality can be either good with probability \( \lambda \) or bad with probability \( (1 - \lambda) \). Regardless of the quality a project can succeed or fail. Good projects succeed with probability \( p_g \in [0, 1] \) and bad projects succeeds with \( p_b \in [0, 1] \), where \( 0 \leq p_b < p_g < 1 \). If a project succeeds it pays off \( I \), and if it fails it pays off \( r = 0 \).\(^3\) By keeping \( I \) constant and letting \( c \) vary, we create a range of different returns, \( I/c \).\(^4\)

There are \( i = 2 \) CRAs able to retrieve a private signal, \( \theta \in (G, B) \), about the true state of the project, \( \omega \in (G, B) \), indicating whether it will be of a good type or a bad type. The rating technology is assumed perfect, corresponding to \( Pr(\theta = G|\omega = G) = e = 1 \) in the literature.

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\(^3\)\( r \) can be thought of as the recovery rate, i.e. the amount recovered in case of default. If \( r > 0 \) the complexity of the analysis would increase, but the outcome wouldn’t.

\(^4\)The same result could have been obtained by letting \( I \) vary and keeping \( c \) constant, so for simplicity we do not vary both variables.
Given the private signal, the CRA can choose to tell the truth or lie. Truth entails always giving a good rating to a good project and a bad rating to a bad project. Lying implies giving a good rating to a bad project or a bad rating to a good project. We distinguish between two types of CRAs:

A. Committed truthful type

B. Opportunistic type

The committed truthful type will always provide a truthful rating, whilst the opportunistic type will try to maximise its payoff and thus may choose to lie.

Since market participants are uncertain regarding the type of the CRA, they assign a posterior probability, $q$, that the CRA is of the truthful type. $q$ is so to speak the reputation of the CRA. A special feature of the credit rating industry becomes important here, as rating agencies are not held legally liable for their ratings, the only punishment from lying, i.e. intentionally providing a wrong rating, is a decrease in the reputation of the CRA.

After retrieving the signal the CRA creates a report, $m \in (g, b)$, indicating the quality of the project. Having observed the report the issuer decides whether to purchase and publish it or not purchase it altogether. If and only if the report is purchased, the CRA receives a fixed fee of $\phi$. This CRA compensation scheme is known as the issuer-pay model.

The opportunistic CRA is faced with the decision of choosing $m$ given $\theta$, this becomes the strategy, $x \in [0, 1]$, of the CRA, which is a mapping of $\theta : \{G, B\} \mapsto m : \{g, b\}$. The CRA chooses the probability of which it untruthfully reports $m = g|\theta = B$, i.e. $x = 1$ implies always giving a good rating to a bad project and $x = 0$ implies always reporting the truth, note a committed truthful CRA will always play $x = 0$.

We assume a bad project is never worth financing, as an investor cannot, in expectation, recoup his investment, i.e. $c > Ip$. An investor will therefore never invest in a project with a bad rating, whereas he may finance a project with good rating, given he deems the rating credible.

An issuer will never be interested in publishing a bad rating as he will have to pay the fee for the rating without being able to receive financing for his project from the investors. This creates a situation where the incentives of the CRAs, issuers and investors are not aligned. The investors

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5 The primary reason for the difference [that rating agencies do not face legal settlements] is that rating agencies have successfully defended themselves against litigation by claiming that their business is financial publishing and that their ratings therefore are "opinions" protected by the First Amendment” Partony (2006)

6 SEC (2008)[p.8] reports that "Typically, the rating agency is paid only if the credit rating is issued, though sometimes it receives a breakup fee for the analytic work undertaken even if the credit rating is not issued"
wish to purchase good quality investments, issuers want to sell their projects and CRAs (if they are opportunistic) wish to maximise profits from issuing good ratings. This misalignment of interests leads to the core issue of the model, namely the following behaviour:

Due the issuer-pay model an opportunistic CRA has an incentive to give a good rating to a bad project in order to collect the issuer-paid fee even though the project does not merit a good rating.

The CRAs are the only long-lived players in the game. Their payoff is the sum of the payoffs from each period discounted by the rate $\delta \in [0,1]$. Both the investors and issuers only live for one period and therefore only play once; this ensures that neither the issuers nor investors play just to gain information regarding the type of the CRA. However the investors and issuers are able to observe the outcomes of previous investments, i.e. they can observe if an investment got a rating, whether it obtained financing, and whether it succeeded or not.

Based on these observations the issuers and investors are able to Bayesian update their beliefs regarding the reputation of the CRA. All previous information in the game is therefore summarised in the Markov state variable, $q$. The updated beliefs are:

$$q^S \equiv \psi(q|S) = \frac{P(q)P(S|q)}{P(S)} = \frac{q}{1 + (1 - q)x(1 - \lambda)\phi \delta}$$ (2.1)

$$q^N \equiv \psi(q|N) = \frac{P(q)P(N|q)}{P(N)} = \frac{q}{1 - x(1 - q)}$$ (2.2)

$$q^F \equiv \psi(q|F) = \frac{P(q)P(F|q)}{P(F)} = \frac{q}{1 + (1 - q)(1 - \lambda)\phi \delta}$$ (2.3)

Where $(S, N, F)$ represents the three outcomes of a project, Success, Failure, and No Financing. With No Financing occurring when a CRA truthfully gives a bad project a bad rating.

The timing of the game is: A cashless firm, which has a project at the cost of $c$ approaches one of two competing CRAs to obtain a credit rating. The approached CRA retrieves a private signal and decides on a rating, according to its strategy $x$. The issuer decides whether to purchase and publish the rating or not. The CRA is paid $\phi$ if and only if the rating is made public. After observing the rating the investors decide whether to invest in the project, taking the reputation, $q$, of the CRA into account. The project is realised and pays off $I$ if successful and 0 if not. The investors and issuers Bayesian update their beliefs regarding the type of the CRA according to the outcome of the project. The game tree is depicted in figure 2.1.

Given this information the investors and issuers are now able to form beliefs regarding the probability of success for a given project, $p(q, x)$. The probability is dependent on the rating
given and the reputation and strategy of the CRA providing the rating.

\[
p(q_i, x_i) = \frac{\text{prob}(\text{success})}{\text{prob}(\text{financing})} = \frac{\lambda p + (1 - q_i)(1 - \lambda) x_i p_b}{1 - (1 - \lambda)(1 - x_i(1 - q_i))}
\]  

(2.4)

2.2 The demand for credit ratings

We assume that investors are risk-neutral, therefore requiring a return, \( R \), which in expectation will let them recover exactly their investment of \( c \). \( R \) can be seen as the interest payment on a loan of \( c \). The investors zero profit condition thus depends on their perceived probability of success given a good rating. Since the probability of success is positively correlated with the reputation \( q \) the interest rate \( R \) decreases as \( q \) increases.

\[
p(q, x) E[R] \geq c
\]

(2.5)

\[
R(q, x) \equiv E[R] \geq \frac{c}{p(q, x)}
\]

It is noted that \( R(q, x) \in [c, I] \), since when \( p(q, x) \to 1 \) then \( R(q, x) \to c \). If \( R > I \) the required return is too large to be covered by the return on the project, and the project will thus not obtain financing.

The issuer obviously is interested in paying the lowest possible interest rate. However from studying the ratings market it appears that issuers rarely change rating agency indicating that pure price competition is not the case. To capture this feature we have modelled the demand as \textit{Differentiated Products Bertrand} known from the Industrial Organisation literature\footnote{Could have chosen otherwise, see section xxx.}.
demand for a rating from $CRA_i$ given there are $n$ competing CRAs on the market is given by:

$$D_i(R_i(q_i, x_i), R_j(q_j, x_j)) = a - bR_i(q_i, x_i) + d \sum_{j \neq i} R_j(q_j, x_j) \quad (2.6)$$

The rating from $CRA_i$ is more attractive the better its reputation is relative to that of the competing $CRA_j$. The demand for a rating from $CRA_i$ is therefore positively correlated with his own $q_i$ and negatively correlated with his competitors $q_j$.

To ensure that all market participants have full information on the effects on next periods reputation of approaching $CRA_i$ in the given period, $CRA_j$ who is not approached for a rating, maintains its current reputation into the next period.

2.3 Sharing the market for credit ratings

The probability that a given CRA is approached is the important parameter in the model, while the demand itself is not. Depending on the cost of the project the possibility of $CRA_i$ being approached is given by:

$$\Phi_i(q_i, q_j) = \begin{cases} \frac{D_i}{\sum D_j} & \text{if } R_i, R_j < I \\ 1 & \text{if } R_{-i} > I > R_i \\ 0 & \text{if } R_i > I \end{cases} \quad (2.7)$$

The probability of being approached for a rating could also be thought of as the market-share in case there were many firms needing financing in each period. The three cases in (2.7) can be illustrated as in figure 2.2 where it, without loss of generality, is assumed that $q_1 > q_2$. The market for credit ratings runs for all projects from $c_L$ to $c_H$. Depending on the reputation of the CRA it will be able to service different deals. We know from (2.5) that if $I > \frac{c}{p(q)}$ a CRA cannot service the deal, thus depending on their respective reputation the CRAs can service deals up until the cut-off points, $s_i = c/I$, at which a CRA falls for the constraints in (2.7).

![Figure 2.2: Illustration of how market is divided between two competing CRAs](Image)

In the illustration $CRA_1$ is the market leader with the highest $q$ and can therefore service the most of the market from $\frac{c}{p}$ to $s_1$. If a $c/I$ less than $s_2$ is drawn both CRAs can service the deal and they split the market according to their respective $\Phi_i$, from the first case of (2.7). For
project between $s_2$ to $s_1$ CRA\textsubscript{1} has the the market to itself, following the second case of (2.7). The final case of (2.7), when projects are more expensive that $s_1$ no CRA can provide a rating.

The way we model market share implies that reputation has two effects on future income. First, the higher $q$ is, the greater is the probability of being market leader and thereby being able to service the market between $s_1$ and $s_2$ alone. Second, if the normalised cost of the project should lay in the range between $s_2$ and $c_L$ then, all else equal, a higher $q$ will raise the probability of being approached for a rating (i.e. the light grey area will be larger than the dark grey, between $c_L/I$ and $s_2$). Thus maintaining a good reputation becomes increasingly important in our model compared to the model of Camanho et al. (2011).

As $c$ is uniformly distributed it is straight forward to calculate the probability that either CRA\textsubscript{1} or CRA\textsubscript{2} will be approached in the given period, for $q_1 > q_2$.

\[
\text{Prob(\text{project comes to CRA}_1)} = \frac{(s_1 - s_2) + \Phi_1(s_2 - \frac{c_L}{\lambda p})}{\frac{c_L}{\lambda p} - \frac{c_H}{\lambda p}}
\]

\[
\text{Prob(\text{project comes to CRA}_2)} = \frac{\Phi_2(s_2 - \frac{c_L}{\lambda p})}{\frac{c_L}{\lambda p} - \frac{c_H}{\lambda p}}
\]

Any project where $c_L \leq (\lambda p_y + (1 - \lambda)p_b)I$ does not need a rating to be financed, since it always fulfils the zero profit condition in (2.5) regardless of $q$. Likewise, any projects where $c_H > p_g I$ will never receive financing as the zero profit condition in (2.5) will be greater than $I$ even when $q = 1$. This yields two boundaries, which allow us to redefine the probabilities that an issuer approaches either CRA:

\[
f_1(q_1, q_2, x_1, x_2) = \frac{(p(q_1) - p(q_2)) + \Phi_1(p(q_2) - (\lambda p_y + (1 - \lambda)p_b))}{p_g - (\lambda p_y + (1 - \lambda)p_b)}
\]

\[
f_2(q_1, q_2, x_1, x_2) = \frac{\Phi_2(p(q_2) - (\lambda p_y + (1 - \lambda)p_b))}{p_g - (\lambda p_y + (1 - \lambda)p_b)}
\]

Note these probabilities do not depend on the size of $I$ and $c$.

### 2.4 Value function of an opportunistic CRA

Given the reputation of the CRA in the beginning of the current period, the opportunist CRA will like to maximise its continuation payoff expressed as the value function, $V_t$, subject to the Bayesian updated reputations given by (2.1)-(2.3).

\[
V_t(q_1, q_2) = f_1(q_1, q_2)\{ \lambda[\phi + p_q \delta V_{t+1}(q_1^S, q_2) + (1 - p_q)\delta V_{t+1}(q_1^F, q_2)] + (1 - \lambda)
\]

\[
[ x_1(q_1, q_2)(\phi + p_q \delta V_{t+1}(q_1^S, q_2) + (1 - p_q)\delta V_{t+1}(q_1^F, q_2))(1 - x_1(q_1, q_2)\delta V_{t+1}(q_1^N, q_2))]
\]

\[
+ f_2(q_1, q_2)\{ \lambda[\phi + p_q \delta V_{t+1}(q_1, q_2^S) + (1 - p_q)\delta V_{t+1}(q_1, q_2^F)] + (1 - \lambda)
\]

\[
[ x_2(q_1, q_2)(\phi + p_q \delta V_{t+1}(q_1, q_2^S) + (1 - p_q)\delta V_{t+1}(q_1, q_2^F)(1 - x_2(q_1, q_2)\delta V_{t+1}(q_1, q_2^N))]
\]

\[
+(1 - f_1(q_1, q_2, x_1, x_2) - f_2(q_1, q_2, x_1, x_2))\delta V_{t+1}(q_1, q_2)
\]

(2.10)
Where \( f_i(q_1, x_1, q_2, x_2) \) is the probability that \( CRA_i \) is approached. However, all market participants are rational, thus able to realise that an opportunistic CRA cannot credibly commit to a such a strategy, \( x \), which maximises the above continuation payoff. Since once faced with a bad project the CRA will find it profitable to deviate and adopt a new rating strategy, \( x \), which maximises:

\[
\max_{x^t \in [0, 1]} V_t(q_1, q_2) = x^t_1(q_1, q_2) \left\{ \phi + p_b \delta V_{t+1}(q_1^S, q_2) + (1 - p_b) \delta V_{t+1}(q_1^F, q_2) \right\} + \left( 1 - x^t_1(q_1, q_2) \right) \delta V_{t+1}(q_1^N, q_2)
\]  

(2.11)

Where \( x^t_1 \) is the rating strategy of \( CRA_1 \) in period \( t \) faced with a bad project.

2.5 Equilibrium properties

All previous actions and outcomes in all previous periods are summarised in the Markov state variable, as described in (2.1)-(2.3), the only ”memory” of previous periods is thus contained in the reputation \( q \), an equilibrium can therefore by described as follows:

**Equilibrium 2.1.** A Markov perfect equilibrium is an equilibrium where:

A. \( x(q_i) \) maximises the continuation payoff in (2.11)

B. Satisfies the probability of success in (2.4)

C. The updated beliefs are (2.1)-(2.3)

After having defined the equilibrium we use (2.11) to derive two properties following Camanho et al. (2011), noting that \( V(q_1, q_2) \) is increasing in \( q_1 \).

**Proposition 2.1.** An opportunistic CRA will always give a good rating to a good project.

*Proof.* See appendix A.1.1

Since an opportunistic CRA always will give a good rating to a good project and a truthful CRA also will, the strategy of either type will always be \( x : \{G\} \rightarrow \{g\} \) following the signal \( \omega = G \). We can therefore solely focus on the properties of \( x \) when the CRA is faced with a bad project, i.e. when \( \omega = B \).

**Proposition 2.2.** If \( V_t(q_1, q_2) \) is increasing in \( q_1 \), there exists a unique \( x_1 \) with 0 \( \leq \) \( x_1 \) \( \leq \) 1.

*Proof.* See appendix A.1.2

It is assumed that as long as \( V(q) > 0 \) a CRA can stay in the game, i.e. even if a CRA is not approached for a rating in the given period it remains in the game by having a chance of being approached for a rating next period.
Proposition 2.3. \( V_i(0, q_j) = 0 \),

From which it follows that if a CRA gets \( q = 0 \) it is eliminated from the game.

Proof. See appendix A.1.3

It is unfortunately not easy to find a strategy \( x \) that satisfies equilibrium 2.1, due to the fact that there does not appear to exist a model in the current literature on a reputation duopoly models with an infinite horizon\(^8\). We therefore proceed to make some simplifying assumptions in order to come up with a closed form solution for a 3-period finite horizon model. Proceeding the finite horizon solution we will examine a dynamic numerical simulation of the infinite horizon model in section 4.

3 Finite horizon solution

We consider a finite 3-period model, in which we set \( p_b = 0 \). Furthermore we assume that the fee \( \phi \) is included in the \( c \).\(^9\) These two adjustments simplifies the analysis greatly. The perceived possibility of success in (2.4) becomes:

\[
p(q_i, x_i) = \frac{\lambda p_g}{\lambda + (1 - \lambda)(1 - q_i)x_i} \tag{3.1}
\]

Furthermore the we can redefine \( c_L = \lambda p_G I \) and \( c_H = p_G I \). Combining with the above we can redefine the probability that a given CRA is approached for a rating, so that (2.8) and (2.9) becomes:

\[
Prob(CRA_1) = \frac{(p(q_1) - p(q_2)) + \Phi_1(p(q_2) - \lambda p_g)}{p_g(1 - \lambda)} \tag{3.2}
\]

\[
Prob(CRA_2) = \frac{\Phi_2(p(q_2) - \lambda p_g)}{p_g(1 - \lambda)} \tag{3.3}
\]

Apart from this the model remains the same although the objective function of the CRA is slightly simplified:

\[
\max_{x_1 \in [0,1]} V_t(q_1, q_2) = x_1(q_1, q_2) \left( \phi + \delta V_{t+1}(q_1^F, q_2) \right) + (1 - x_1(q_1, q_2)) \delta V_{t+1}(q_1^N, q_2) \tag{3.4}
\]

These simplifying assumptions make no difference for proposition 1.1 and 1.2. By using proposition 1.2 and realising the (3.4) is linear in \( x_1(q_1, q_2) \), it follows that the solution to the maximisation problem in (3.4) must be unique and existing.

Proposition 3.1. An opportunistic CRA will always have an equilibrium strategy \( x > 0 \) given that \( p_G < 1 \)

\(^8\)See Bar-Isaac and Tadelis (2008)

\(^9\)i.e. we define \( c' = c + \phi \), but for simplicity we use the notation \( c \) instead
Proof. See appendix A.1.4

In order to solve the finite horizon, a terminal condition of the model is needed.

**Proposition 3.2.** If the game ends in period \( T \), an opportunistic CRA will always play \( x = 1 \) in period \( T \) and \( T - 1 \)

Proof. See appendix A.1.5

By using proposition 1.5 we derive an analytical solution to \( x \) in period \( T - 2 \), we do so by first looking at the monopoly case.

### 3.1 Monopoly in period T-2

By using proposition 1.3 we know, that if \( q = 0 \) a CRA will never be approached. So by solving the model for \( 1 \geq q_i > 0 \) and \( q_j = 0 \) we can derive a solution for the monopoly case, under the simplifying assumption that \( p_g = 1 \) and \( p_b = 0 \).

**Proposition 3.3.** A monopoly CRA will in period \( T-2 \), where the game ends in period \( T \) play the following strategy:

\[
x_i(q_i, 0) = \begin{cases} 
0 & \text{if } A \leq \frac{\lambda q}{\lambda q + (1-q)} \\
1 - \frac{(1-A)\lambda q}{A(1-q)} & \text{if } \frac{\lambda q}{\lambda q + (1-q)} < A < 1 \\
1 & \text{if } A = 1
\end{cases}
\]

(3.5)

Where \( A \) is the solution to \( \delta^2(1 - \lambda)A^2 - (\delta + \delta^2)A + 1 = 0 \)

Proof. See appendix A.1.6

From the proposition above we see that \( x \) is decreasing in \( q \), since \( \frac{(1-A)\lambda q}{A(1-q)} \) is increasing in \( q \), all else equal. The result that the higher reputation a CRA has the less it is likely to lie, arises from the fact that we have assumed \( p_g = 1 \). The assumption implies that a ”lying” CRA will be caught immediately, which will cause its \( q \) to fall to 0, following proposition 1.3 this results in the CRA gaining 0 profit in all remaining periods.

Thus when a CRA has a low \( q \) the possibility of being approached for a rating in the future is low, making it more profitable to lie in the current period. Contrary when a CRA has a high \( q \) the possibility of attracting business in the future is high, making lying more costly.

\( x \) is decreasing in \( q \) until \( q = 1 \) is reached, where \( x = 1 \). This result arises from the fact that \( q = 1 \) can only be achieved if the CRA type-reveals itself as the truthful type, making the analysis irrelevant as a truthful CRA always plays \( x = 0 \), i.e. if the CRA is an opportunistic
type it can never obtain a \( q = 1 \). Under the model assumptions a monopolistic CRA will never lie for sure, but it will lie with a very high probability when the reputation approaches 0.

However our results hinges on a few key assumptions namely: The 3-period setting limits the value of a high reputation as there are only two proceeding fees to worry about, and \( p_g = 1 \) and \( p_b = 0 \) results in immediate type-revealing in case of a failure, increasing the cost of lying.

3.2 Duopoly in period T-2

In the duopoly case we assume \( p_G = 1, p_B = 0 \) and \( b = d \) in the demand function (2.6). Furthermore to ensure that \( D(q_i) > 0 \ \forall \ q \), we use that \( R = \frac{c}{p(q,x)} \) and knowledge of the lowest possible \( p(q,x) = \lambda \) and highest possible \( p(q,x) = 1 \), we assume \( a(c) = \frac{c(1-\lambda)}{\lambda} \) in the demand function. Using this we model the probability of being approached as:

\[
\Phi_i | R_{i,j} < I = \frac{D_i}{D_i + D_j} = \frac{a(c) - R_i + R_j}{2a(c)} = \frac{\frac{c(1-\lambda)}{\lambda} - \frac{c}{p(q_i,x_i)} + \frac{c}{p(q_j,x_j)}}{2\frac{c(1-\lambda)}{\lambda}} \tag{3.6}
\]

Given a project with a cost low enough, so as both CRAs can be approached, the market is shared as in the expression above. If one of the CRA cannot attract the project due to a too low \( q \), the issuer will go to the other CRA with certainty. If neither of the CRAs have a good enough reputation, none of them are approached. The result from proposition 1.5, showing that an opportunistic CRA always will lie in the two last periods still holds, which leads to the following decision rule, which is a main finding in our thesis (and we would like to direct the attention of interested reader to appendix 1.7):

**Proposition 3.4.** A duopolistic CRA will in period T-2, where the game ends in period T adopt the following rating strategy:

\[
x_1(q_1, q_2) = \begin{cases} 
0 & \text{if } A' \leq \Phi_1 A_1(1-q_1) \\
1 - \frac{(1-A') \lambda q_1}{A(1-q_1)} & \text{if } \Phi_1 \frac{\lambda q_1}{A_1} < A' < \Phi_1 \\
1 & \text{if } A' = \Phi_1 
\end{cases} \tag{3.7}
\]

Where \( A' \) is the solution to

\[
0 = 1 - \left\{ \delta \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) + \delta^2 \left( \lambda \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right)^2 + \left( \frac{1}{\Phi_2} B' - \min(A', B') \Phi_1 \max(\Phi_1, \Phi_2) \right) \left( \lambda \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) + (1-\lambda)(1-q_2) \frac{1}{\Phi_1} A' \right) + (1-\lambda) q_2 A'_{q_2=1} \right) + (1-\max(A', B') \min(\frac{1}{\Phi_1}, \frac{1}{\Phi_2}) \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) \right) \right\}
\]

and \( B' = \Phi_2(q_2, 1, q_1 N, 1) \frac{p(q_2)-cH/I}{cH/I-cL/I} \)

**Proof.** See appendix A.1.7
Like in the monopoly case, we have that $x_1$ is decreasing in $q_1$. As $CRA_1$’s reputation increases the probability of being approached in both upcoming periods increases, thereby increasing expected future profits, making lying more costly. It is only in the situation where $CRA_1$ receives both upcoming fees, that he will find it more profitable to stay in the market than simply lie now and take the fee in the current period.

Lying entails a type-revealing failure for sure. Thus $V(q_F)$ remains constant at zero. For the indifference between lying and telling the truth in decision rule (2.11) to hold, $q_N$ must fall so $\phi$ equals $V(q_N)$. For these reasons the CRA will lie a little less for a small increase in reputation.

As the reputation of the competing $CRA_2$ increases, $CRA_2$ is more likely to be approached in one of the following periods, thus lowering the expected future profits of $CRA_1$. Hence $CRA_1$ will find it profitable to inflate ratings compared to increase its reputation from a No Rating outcome. I.e. $CRA_1$ is more likely to take the fee in current period rather than hoping for the now slimmer chance of being approached in the two subsequent periods.

The rating strategies of an opportunistic $CRA_1$ in both the monopolistic and duopolistic 3-period finite horizon setting are graphed in figure 3.1. This is a clear illustration of the analytically found results in propositions 1.6 and 1.710.

![Figure 3.1: Strategy of a strategic duopolistic CRA for different values for competitor CRAs reputation, $q_2$](image)

We found that as $q$ increases, the term $\frac{(1-A)\lambda q}{A(1-q)}$ increases, thereby lowering the associated $x$, this can be seen from the declining shape of the dotted line, which represents the monopoly case, i.e. for a higher reputation the cost of lying goes up as expected future profits increase, and thus the CRA lies less. When $q$ becomes sufficiently high the $A$ will fall below the boundary condition $\frac{\lambda q}{\lambda q + (1-q)}$ making $x = 0$. In the literature this is known as the disciplining effect.

The disciplining effect arises from investors and issuers rationally updated Bayesian beliefs regarding the type of the CRA. The higher the reputation the higher the belief of the CRA being the honest type, which again means a higher probability that the CRA being approached

\[\text{We have verified through multiple parameter values, that this result is robust. Graphs can be e-mailed upon request.}\]
in the following periods. With the chosen parameter values of \( p_g = 1, p_b = 0 \) lying entails a type-revealing failure with the reputation going to zero, making lying even more costly. Note, that the disciplining effect is not strong enough to ensure \( x = 0 \) for all values of \( q \).

The solid lines in figure 3.1 represents the rating strategy for the incumbent \( CRA_1 \) in a duopolistic setting analytically found in proposition 1.7 for different values of \( q_2 \). From this we see an unambiguous effect of competition. The introduction of a competitor \( CRA_2 \) increases the incentive to adopt a less honest rating strategy for the incumbent \( CRA_1 \). The change in strategy arises from two effects; the *disciplining effect* and the *market sharing effect*.

The disciplining effect remains conceptually the same as in the monopoly case. The market sharing effect captures the notion of lost future profits as an increased amount of the market must be shared with a competing \( CRA_2 \). This effect is amplified as the competing \( CRA_2 \)’s reputation increases. This is clearly illustrated in figure 3.1 where the solid line shifts outwards in the \((q_1, x_1)-\)space as \( q_2 \) increases.

When \( q_2 \) increases the investors believe that the competing \( CRA_2 \) is more likely to be of the honest type. Therefore investors require a lower return on projects, rated by the \( CRA_2 \). With the lowered required return \( CRA_2 \) can now facilitate a greater share of the market. In the existing market the competition is intensified through a larger \( \Phi_2 \), this effect is new compared to the model by Camanho et al. (2011). Altogether \( V_{T-2}(\cdot, q_2) \) is lowered when \( q_2 \) increases, making it more profitable to lie and take the fee in the current period. Note, in the event of a ”bad rating” the issuers and investors belief that \( CRA_1 \) is of the honest type is increased as a strategic CRA is expected to lie with a high probability, thus the outcome ”bad rating” has an increased informational value.

The market sharing effect is relative to the incumbent \( CRA_1 \)’s position as a function of \( \Phi_2 \) whereas the market sharing effect is absolute in \( q_2 \) in the model by Camanho et al. (2011). Therefore reputation becomes increasingly important in our model. Since the effect from increased competition is unambiguous for all possible reputation combinations, the market sharing effect must be overpowering the disciplining effect, causing the incumbent \( CRA_1 \) to inflate ratings in a higher percentage of the cases.

### 4 Dynamic numerical simulation of the infinite horizon

As we cannot derive a closed for solution to the Markov Perfect Equilibrium for the infinite horizon, we therefore present a numerical solution. Exploiting Denardo’s contraction mapping
a finite horizon dynamic programming solution of the CRAs strategy will approach the infinite horizon strategy and converge on a unique fixed point for any given reputation. We use the results derived in (1.4) and (1.5) for the finite horizon model to find the fixed terminal condition for the last period, $T$, and second last period, $T-1$, i.e. $x_0 = x_T(q_1, q_2) = 1$. Proceeding by backwards induction to solve one period at a time, we increase $T$. When $T$ increases towards infinity it allows us to drop the subscript $t$ on our value function in (2.10) and decision function (2.11) giving us:

$$V_1(q_1, q_2) = \begin{cases}
  i_{q_1 > q_2} [(P_1-P_2)+\Phi_1(P_2-\lambda p_g)] + i_{q_1 \leq q_2} \Phi_1(P_2-\lambda p_g) \\
  + (1-\lambda) \left[ \lambda \left[ \phi + p_g \right] \right] + (1-\lambda) \left[ x_1(q_1, q_2) \left( \phi + p_g \delta V(q_1^{S}, q_2) + (1-p_b) \delta V(q_1^{S}, q_2) \right) \right] + (1-x_1(q_1, q_2) \delta V(q_1^{N}, q_2)) \right]
  + \left( 1 \lambda \delta V(q_1^{N}, q_2) + (1-p_g) \delta V(q_1^{S}, q_2) + (1-\lambda) \left[ (1-q_2) x_2(q_1, q_2) (p_b \delta V(q_1, q_2) + (1-p_b) \right] \right) \delta V(q_1, q_2)^N \right] + \frac{1-p_1(q_1 > q_2) - P_2(q_1 \leq q_2)}{1-\lambda p_g} \delta V(q_1, q_2) \\
\end{cases}$$

Where $i_{q_1 > q_2}$ is the probability of the issuer approaching CRA1 for a rating, $i_{q_1 \leq q_2}$ is the probability CRA2 is approached and $1-p_1(q_1 > q_2) - P_2(q_1 \leq q_2)$ is the probability that the project is too expensive and thus cannot be rated by either CRA.

This allows us to compare our model with the Duopoly case of Camanho et al. (2011) for various parameters. Mail authors for MatLab code. We show,

i) Reputation is not enough to discipline the opportunistic monopoly CRA

ii) Competition is not sufficient to make opportunistic CRAs act as if honest

iii) Competition will for most instances aggravate the problems of rating inflation

iv) Comparing with Camanho et al. (2011) we find competition less problematic

### 4.1 Monopolist CRA

Considering the case for a monopoly CRA our model coincides with the one by Camanho et al. (2011). As described in section 3.1, monopoly for CRA1 arises when $q_2 = 0$.

Figure 4.1 plots the strategy of a strategic monopolistic CRA as a function of its beginning of period reputation, when approached by an issuer with bad-type project. The function is

11Judd (1998) [Ch. 12 Numerical Dynamic Programming, sub Infinite-Horizon Problems p. 401-403]
12Parameter values are chosen to make it comparable to Camanho et al. (2011) [figure 3]
13Following proposition 1.1 a CRA faced with a good-type will always play $x = 0$
"U-shaped" and strictly positive for all $q$’s. Clearly reputational concerns are not sufficiently strong to ensure that an opportunistic CRA will behave honestly faced with a bad-type project.

Intuitively this shape arises from the trade-off between current period income and future income. When $q$ is low the possibility of being approached for a rating in the future is low, thus expected future income is low. The opportunistic CRA faced with a bad project will lie with a higher possibility to squeeze the last dollar before going out of business, i.e if $q = 0.05$ the CRA will lie in roughly $60\%$ of the times approached. Note a high $x$ also increases the reputation gain of not rating. As $q$ increases its expected future income also increases, making it profitable to play a lower $x$ to build an even higher reputation. If the $q$ becomes very high the strategic CRA will begin to lie with a high probability again. This occurs because market participants are more likely to attribute failures to bad luck rather than rating inflation. Thus the CRA is able to "cash-in" on its good reputation.

Figure 4.1: Strategy of a strategic monopolistic CRA $(\lambda, p_g, \delta, p_b) = (0.5, 0.7, 0.9, 0.0)$

Figure 4.2: Strategy of monopolist CRA for different values of $p_g$, $p_b$ and $\lambda$

Figure 4.2 plots the strategy of a strategic monopolistic CRA for different parameter values.
In panel 4.2(a) we see that $x$ is decreasing in $p_g$, i.e. $\frac{\partial x}{\partial p_g} < 0$. The reason being the reputational concern for the CRA captured in (2.1)-(2.3). $q^F$ and $q^S$ are decreasing in $p_g$ and $q^N$ is unaffected by $p_g$. Thus the reputational penalty from lying is increasing in $p_g$ because when $p_g$ is high an eventual failure is more likely to stem from rating inflation than from bad luck, which in equilibrium makes the strategic CRA play a lower $x$ the higher $p_g$ is.

A similar story is told in panel 4.2(c) if projects are less likely to be good, i.e. $\lambda$ is low, then failures are less likely to be attributed to bad luck, increasing the reputational penalty, making it less profitable to inflate ratings, i.e. $\frac{\partial x}{\partial \lambda} > 0$. In panel 4.2(b) it can be seen that $x$ is increasing in $p_b$, i.e. $\frac{\partial x}{\partial p_b} > 0$, the story is similar to the two above. When $p_b$ increases a failure is less likely to be attributed to ratings inflation thus the reputational penalty becomes less making it more profitable to lie.

### 4.2 Duopolist CRA

The duopoly case allows us to examine the effects of competition. The strategy and value function of an opportunistic $CRA_1$ changes with its own reputation and that of its competitor, $CRA_2$.

![Figure 4.3: Rating policy for $CRA_1$ as function for $q_1$ and $q_2$ ($\lambda, p_g, \delta, p_b$) = (0.5, 0.7, 0.9, 0.0)](image-url)
Figure 4.3 plots the optimal rating strategy for an opportunistic CRA as a function of \((q_1, q_2)\). Figures 4.4 and 4.5 plot cross-sections of 4.3 for a fixed values of \(q_2\) and \(q_1\) respectively.

Figure 4.4 plots the relationship between the rating policy of CRA1 and its own reputation, which remains a U-shape as in the monopoly case. We can therefore conclude competition does not prevent rating inflation, i.e. \(x_1 > 0\). As the competing CRA2’s reputation increases, going from panel (a) to (c), we see the curve shifts upwards, so as the opportunistic CRA1 will find it more profitable to lie. The minimum value for \(x_1\) shifts along the horizontal axis as the fixed value of \(q_2\) increases.

\[ \begin{align*}
\text{(a)} & \quad q_2 = 0.25 \\
\text{(b)} & \quad q_2 = 0.55 \\
\text{(c)} & \quad q_2 = 0.75
\end{align*} \]

**Figure 4.4:** Rating policy for CRA1 for fixed values of \(q_2\) \((\lambda, p_g, \delta, p_b) = (0.5, 0.7, 0.9, 0.0)\)

The greater the distance between their respective reputations, the more likely CRA1 is to lie. This behaviour can be attributed to the disciplining effect. When the reputations are close to each other the risk/opportunity of losing/gaining the market leading position is greater, hence an opportunistic CRA will act more cautious. The reverse is true when the CRA is entrenched as either the market leader or bottom dweller, as position changes are not likely to occur.

\[ \begin{align*}
\text{(a)} & \quad q_1 = 0.25 \\
\text{(b)} & \quad q_1 = 0.55 \\
\text{(c)} & \quad q_1 = 0.75
\end{align*} \]

**Figure 4.5:** Rating policy for CRA1 for fixed values of \(q_1\) \((\lambda, p_g, \delta, p_b) = (0.5, 0.7, 0.9, 0.0)\)

Figure 4.5 plots the relationship between the rating policy of CRA1 and its competitors reputation. It is flat or decreasing for low values of \(q_1\) relative to \(q_2\) and increasing for higher values of \(q_2\). This shape is governed by the market sharing effect and the disciplining effect.

The disciplining effect is described above. The market sharing effect is increasing in \(q_2\),
because the cost of lying decreases as a larger share of the future market must be relinquished to the competitor as his reputation increases. Hence for large values of $q_2$ the market sharing effects tends to dominate the disciplining effect.

![Graphs](image1.png)

**Figure 4.6:** Rating policy for $CRA_1$ for fixed values of $q_2$ compared to the monopoly case. The solid line represents the monopoly and the dashed line the duopoly.

Figure 4.6 compares figure 4.1 and 4.4. For low values of $q_2$ the opportunistic CRA behaves more strict than the monopoly CRA. If the competing CRA is not a too serious threat, competition can work as a disciplining mechanism.

However, as $q_2$ increases, the market sharing effect will begin to dominate the disciplining effect for lower values of $q_1$; as future profits decrease with the lessened likelihood of being approached in the future, i.e. the cost of lying decreases. These results are central to our model as it describes the effects of introducing competition to the market for credit ratings.

As mentioned, we have build on the model by Camanho et al. (2011) and adopt a more competition sensitive assumption regarding how the market is shared. In order to compare the two we have simulated both models. Note, we have not been able to replicate their exact rating strategy graphed in figures 3-12 in Camanho et al. (2011), though the functional shapes are identical. In general our simulation of their model tends to provide slightly lower values for $x$ for the same parameter values.\(^\text{14}\) This however does not change any qualitative conclusions.

We see from figures 4.7 and 4.8 that our modelling translates into a more strict - but not truth telling - rating policy for an opportunistic CRA for practically all values of $q_1$ and $q_2$.

In figure 4.7 we see our model adjustments causes a downwards shift in the strategy up to 0.1 $x$ corresponding to a 10% to 30% decrease in lying. The shape remains the same thus the underlying incentives have not changed, although the disciplining effect is amplified due to our modelling of $\Phi$. The increased competition on the bottom of the market is essentially, what causes the downwards shift.

\(^{14}\)We have tried to contract the authors in order to share codes without any luck.
Figure 4.7: Rating policy for CRA\textsubscript{1} for fixed values of $q_2$. The solid line represents the model by Camanho et al. (2011) and the dashed line our model.

(a) $q_2 = 0.25$
(b) $q_2 = 0.55$
(c) $q_2 = 0.75$

Figure 4.8: Rating policy for CRA\textsubscript{1} for fixed values of $q_1$. The solid line represents the model by Camanho et al. (2011) and the dashed line our model.

(a) $q_1 = 0.25$
(b) $q_1 = 0.55$
(c) $q_1 = 0.75$

Figure 4.8 tells the same story. We see for all but the smallest values of $q_2$ a parallel downwards shift in the rating strategy. The shift is caused by the same effect as described above. However notice that for $q_2 < 0.05$ and increasing $q_1$ the opportunistic CRA\textsubscript{1} in our setting tends to lie more. This happens because the threat of losing a significant market share to CRA\textsubscript{2} becomes too small to have a disciplining effect on CRA\textsubscript{1}, causing an increase in $x_1$.

To sum up, our model shows that by increasing the competition on the bottom market, the disciplining effect becomes larger than in the model set up by Camanho et al. (2011). This could indicate that in a highly competitive market, increased competition could improve ratings quality. To examine this we look at:

$$\text{Excess Lax Behaviour of } CRA_1 = \int_{q_2 \in [0,1]} x_1(q_1, q_2) dq_2 - x_1(q_1, 0)$$  \hspace{1cm} (4.2)

Excess lax behaviour measures the difference in a rating policy between the monopoly and duopoly CRA for all values of competitor reputation. For positive values of excess lax behaviour a duopolistic CRA will lie more than the monopolistic CRA and vice versa. From figure 4.9 we see the excess lax behaviour of our model and that of Camanho et al. (2011). Once again the two have the same functional shape, but our model is shifted down compared to Camanho et al. (2011). For low values of $q_1$ we see a hump stemming from the domination of market sharing.
Figure 4.9: Excess Lax Behaviour arising from competition. The solid line represents the model by Camanho et al. (2011) and the dashed line our model

effect. I.e. for low reputations of the incumbent $CRA_1$, entry will induce a large reduction in market share, future expected profits and therefore the cost of lying decreases thus increasing lax behaviour. As $q_1$ increases the disciplining effect becomes more pronounced. For very high values of $q_1$ failures are likely to be attributed to bad luck rather than rating inflation resulting in an increased lax behaviour.

An interesting feature is the fact that we observe a slight negative excess lax behaviour for $0.45 < q_1 < 0.9$. This indicates that, should an incumbent lie within this interval entry from $CRA_2$ could on average force the incumbent $CRA_1$ to rationally adopt a slightly more strict rating policy (to the tune of $-0.01x$).

5 Conclusion

Our article analyses the credit ratings market and its misalignment of incentives, focusing on competition between CRAs. We have incorporated the influence of payments by issuers, the reputation of CRAs, and rational investors on the capital market.

Based on Camanho et al. (2011) we construct a new framework for the credit rating market under duopolistic competition, making the more realistic assumption that rated firms’ excess profits flow to their shareholders. Our model finds reputational concerns not to be a sufficient disciplining mechanism to ensure an opportunistic CRA will adopt a truthful rating regime, in line with the existing literature. Further we find competition is not sufficient to make opportunistic CRAs act adopt a truthful rating regime. In fact competition will for most instances aggravate the problems of rating inflation.

This has direct implications on policy recommendations. Competition enhancing policy proposals will not push the market equilibrium into the desired state, where CRAs accurately
predict all credit ratings. E.g. the 3 year rotation scheme drafted by EU commission may not be favourable\textsuperscript{15}.

The issuer-pay model inherently aligns the incentives of the CRA with those of the issuer. Competition will never break this misalignment, calling for a structural changes on the market. Such a structural change could be to revert back to the investor-pay model of the early ages of the credit rating market. However, free-riding issues make this option non-feasible as a market-based solution.

The Cuomo plan of upfront payment for non-contingent ratings would solve the misalignment. However cashless firms cannot pay upfront leaving the problem unsolved. The platform-pays model proposed by Mathis et al. (2009) cuts the direct commercial link between issuers and CRAs. In theory it would work in our model, however real life implementation seems unlikely due to the continuous dynamic exchange of information. Increased transparency in the ratings process and flow of fees\textsuperscript{16} will increase the informational content of outcomes producing more favourable equilibria\textsuperscript{17}.

Finally, for certain specifications of our model competition actually decreases lax behaviour. The existing literature suggest two effects of competition, the market sharing effect and the disciplining effect. Our modelling results in a stronger disciplining effect, as competition for the entire market is now affected by reputational concerns. Thus competition aggravates lax behaviour less than earlier suggested according to our model, as reputational competition is increased.

\textsuperscript{15}The EU commission has suggested the no issuer are allowed to keep the same CRA for more than three years. See: http://europa.eu/rapid/pressReleasesAction.do?reference=IP/11/1355

\textsuperscript{16}This could be done by forcing CRAs to publish income-per-client lists or force corporations to disclose detailed rating expenses in their annual reports.

\textsuperscript{17}increasing \( p_y \) and lowering \( p_b \) reduces lax behaviour in our model. This change in parameters approaches perfect observation of the CRAs by the investors.
A Appendix

A.1 Proofs

A.1.1 Proof of proposition 1.1

Proposition 1.1. An opportunistic CRA will always give a good rating to a good project.

Proof. If we assume that an opportunistic CRA is approached for a rating by an issuer with a good project, then the CRA will adopt a rating policy $x_i : \{G, B\} \mapsto \{g, b\}$. We now examine three different rating strategies, $x_i$, and see whether the CRA wants to deviate:

(a) $x_i = 1$

If it is an equilibrium strategy to always lie it must be the case that $V(\text{lie}) \geq V(\text{honest})$. In case of a bad project that means: $\phi + \delta V_{t+1}(q_i^F, q_j) \geq \delta V_{t+1}(q_i^N, q_j)$. The gain from giving a good rating to a good project is: $\phi + \delta p_g V_{t+1}(q_i^S, q_j) + \delta (1-p_g) V_{t+1}(q_i^F, q_j)$ whereas the gain from giving it a bad rating is: $V_{t+1}(q_i^N, q_j)$. As $q^S > q^F$ and $V(q)$ is increasing in $q$ this must be larger than $\phi + \delta V_{t+1}(q_i^F, q_j) \geq \phi + \delta V_{t+1}(q_i^N, q_j)$, thus the CRA does not want to deviate.

(b) $x_i = 0$

If it is an equilibrium strategy to always report the truth, then according to (2.1)-(2.3) there is no gain in reputation regardless of the outcome. An opportunistic CRA therefore has no incentive to forego the fee by giving a good project a bad rating as there is no gain in reputation tomorrow.

(c) $0 < x_i < 1$

If it is an equilibrium strategy to lie with a positive probability, we must have that $V(\text{lie}) = V(\text{honest})$. In case of a bad project that means: $\phi + \delta V_{t+1}(q_i^F, q_j) = \delta V_{t+1}(q_i^N, q_j)$. The gain from giving a good rating to a good project is: $\phi + \delta p_g V_{t+1}(q_i^S, q_j) + \delta (1-p_g) V_{t+1}(q_i^F, q_j)$. As $q^S > q^F$ and $V(q)$ is increasing in $q$ this must be larger than $\phi + \delta V_{t+1}(q_i^F, q_j) = \delta V_{t+1}(q_i^N, q_j)$. Thus the CRA does not want to deviate.

The CRA will therefore always give a good rating to a good project.

A.1.2 Proof of proposition 1.2

Proposition 1.2. If $V_t(q_1, q_2)$ is increasing in $q_1$, then there exists a unique $x_1$, where $0 \leq x_1 \leq 1$. 

25
Proof. We know that if:

(a) If the equilibrium strategy is \( x_1 = 1 \), we must have that \( V(\text{lie}) > V(\text{honest}) \)

(b) If the equilibrium strategy is \( x_1 = 0 \), we must have that \( V(\text{lie}) < V(\text{honest}) \)

(c) If the equilibrium strategy is \( 0 < x_1 < 1 \), we must have that \( V(\text{lie}) = V(\text{honest}) \)

If a CRA is faced with a good project it will play \( m = g \) following proposition 1.1. However faced with a bad project the pay-off is: \( V(\text{lie}) = \phi + \delta V_{t+1}(q_i^F, q_j) \) if it lies and \( V(\text{honest}) = V_{t+1}(q_i^N, q_j) \) if it tells the truth.

From (2.3) and (2.2) we have that: \( q^F = \frac{q}{1+(1-\varphi)(1-\lambda)p_B/(1-\varphi)} \) and that \( q^N = \frac{q}{1-x(1-\varphi)} \), from these two equation it is easy to see that \( q^F \) is decreasing in \( x \) and \( q^N \) is increasing in \( x \). Given that \( V_t(q_i, q_j) \) is increasing in \( q_i \) we have that \( V(\text{lie}) \) is decreasing in \( x_i \) and that \( V(\text{honest}) \) is increasing in \( x_i \). Therefore the rating strategy, \( x \), must be well-defined and unique.

A.1.3 Proof of proposition 1.3

Proposition 1.3. \( V_i(0, q_j) = 0 \),

From which it follows that if a CRA gets \( q = 0 \) it is eliminated from the game.

Proof. If \( q = 0 \) then by (2.1)-(2.3) \( q^S = q^N = q^F = 0 \). There is no way for the CRA to improve its reputation once \( q \) has hit 0. Thus there is no reason to deviate from \( x = 1 \). From (2.4) we get that \( p(0, 1) = \lambda p_G + (1-\lambda)p_B \) which is the a priori belief of success. If we assume that the cheapest possible project is drawn we have from (2.5) that \( I > R(q) = \frac{c_L}{p(0, 1)} = \frac{i(\lambda p_G + (1-\lambda)p_B)}{\lambda p_B + (1-\lambda)p_G} = I \) which obviously cannot be true. No issuer will therefore approach a CRA with \( q = 0 \), hence the CRA will not be able to continue.

A.1.4 Proof of proposition 1.4

Proposition 1.4. An opportunistic CRA will always have an equilibrium strategy \( x > 0 \) given that \( p_G < 1 \)

Proof. We know that if \( x = 0 \) and \( p_g = 1 \) then reputitional concerns becomes irrelevant as \( q^N = q^F = q^S = q \) following (2.1)-(2.3). So for \( x = 0 \) to be an equilibrium strategy it must be that \( V(\text{honest}) > V(\text{lie}) \) which in case of a bad project means that: \( \phi + \delta V_{t+1}(q_i^F, q_j) < \delta V_{t+1}(q_i^N, q_j) \). But as long as \( q^N = q^F \) this cannot be the case, given that \( \phi > 0 \), thus \( x = 0 \) can never be the equilibrium strategy of on opportunistic CRA.
A.1.5 Proof of proposition 1.5

Proposition 1.5. If the model ends in period $T$, then an opportunistic will always play $x = 1$ in period $T$ and $T-1$

Proof. Considering the strategy in each of the following two periods:

Period T As the game finishes in period T, the opportunistic CRA has no incentive to tell the truth, given it is approached with a bad project, as the trade-off is between $\phi$ today and nothing tomorrow.

Period T-1 In the second last period if the CRA chooses to lie it will receive $\phi + \delta f(q^F, 1, q_j, 1)\phi$ as we know that an opportunistic CRA will play $x = 1$ in period T. If it chooses to tell the truth it will receive $\delta f(q^N, 1, q_j, 1)\phi$. We know that $\delta < 1$ and that $f(q_i, x_i, q_j, x_j) \leq 1$. Therefore the gain from telling the truth must, even in the case where $\delta f(q^F, 1, q_j, 1)\phi = 0$, be less than the gain from lying. Meaning that no repetitional concern is big enough to abstain an opportunistic CRA from lying in the second last period.

An opportunistic CRA will therefore always play $x = 1$ in period T and T-1. \qed

A.1.6 Proof of proposition 1.6

Proposition 1.6. A monopoly CRA will in period T-2, where the game ends in period T play the following strategy:

$$x_i(q_i, 0) = \begin{cases} 0 & \text{if } A \leq \frac{\lambda q}{\lambda q + (1-q)} \\ 1 - \frac{(1-A)\lambda q}{A(1-q)} & \text{if } \frac{\lambda q}{\lambda q + (1-q)} < A < 1 \\ 1 & \text{if } A = 1 \end{cases}$$  \quad (A.1)

Where $A$ is the solution to $\delta^2(1-\lambda)A^2 - (\delta + \delta^2)A + 1 = 0$

Proof. If $q_j = 0$ we have a monopoly case following proposition 1.3. Thus the probability of the project coming to $CRA_i$ becomes:

$$f_M(q_M) = \frac{p(q_M) - \frac{\lambda p}{2}}{\frac{\lambda p}{2} - \frac{\lambda p}{2}} = \frac{p(q_M) - (\lambda p_G + (1-\lambda)p_B)}{(1-\lambda)(p_G - p_B)}$$  \quad (A.2)

where $q_M$ indicates that $1 \geq q_i > 0$ and $q_j = 0$. We know that if the CRA is approached with a good project it will always play $m = g$ following proposition 1.1, in the proof we at the behaviour when an opportunist CRA is approached with a bad project. If $V(lie) > V(honest)$ the CRA will lie. If $V(lie) < V(honest)$ it will be honest. If $V(lie) = V(honest)$ it will adopt a rating strategy where $0 < x < 1$. 

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We make the simplifying assumption that: \( p_G = 1 \), \( p_B = 0 \) which entails (2.3) that \( q^F = 0 \) and \( q^S = q \). By using proposition 1.5 it must therefore be the cases that

\[
V(lie) = V(q_M, x_i = 1) = \phi \\
V(honest) = 0 + \delta V_{t+1}(q_M^N, x_i = 1) = \delta \phi f_2(q_M^N, 1) + \delta^2 \phi(1 - (1 - \lambda)f_2(q_M^N, 1))f_3(q_M^N, 1)
\]

We therefore examine the case where \( V(lie) = V(honest) \),

\[
\frac{1}{V(lie)} - \frac{\delta f(q_M, 1) - \delta^2 (1 - (1 - \lambda)f(q_M, 1))f(q_M, 1)}{V(honest)} = 0 \tag{A.3}
\]

we then define \( A \equiv f(q_M^N, 1) \leq 1 \), simplifying the above to a second degree polynomial.

\[
\frac{\delta^2 (1 - \lambda)}{a} A^2 - \left(\delta + \delta^2\right) A + \frac{1}{c} = 0
\]

which yields

\[
A = \begin{cases} \\
\frac{\delta + \delta^2 - \rho_M}{2\delta(1 - \lambda)} \\
\frac{\delta + \delta^2 + \rho_M}{2\delta(1 - \lambda)}
\end{cases}
\]

where \( \rho_M = \sqrt{(\delta + \delta^2)^2 - 4\delta^2(1 - \lambda)} \geq 0 \). We see that if \( \delta \geq 2\sqrt{1 - \lambda} - 1 \) is satisfied there always exists a solution to \( A \).

Knowing an opportunistic CRA will play \( x_{T-1} = x_T = 1 \) and the assumptions of \( p_G = 1p_B = 0 \) we simplify (A.2):

\[
f_M(q_M) = \frac{\lambda q^N}{\lambda q^N + 1 - q^N} \tag{A.4}
\]

By using \( q^N = \frac{q}{1 - x(1 - q)} \) we then solve

\[
A \equiv f(q_M^N, x_{T-2}, x_{T-2}, x_{T-1} = 1) \text{ for } x_{T-2} \\
A = \frac{\lambda q^N}{\lambda q^N + 1 - q^N} \Rightarrow \\
x(1 - q) = 1 - \frac{A}{A(1 - \lambda)} - \frac{\lambda q}{A} \Rightarrow \\
x = 1 - \frac{(1 - A)\lambda q}{A(1 - q)}
\]
We then solve the above for $x \in [0,1]$ to find boundaries for $A$.

\[
x > 0 \Rightarrow 1 - \frac{(1 - A)\lambda q}{A(1 - q)} > 0 \Rightarrow \frac{(1 - A)\lambda q}{A(1 - q)} < 1
\]

\[
(1 - A)\lambda q < A(1 - q) \quad \text{and} \quad A > \frac{\lambda q}{1 - (1 - \lambda)q}
\]

\[
x < 1 \Rightarrow 1 > 1 - \frac{(1 - A)\lambda q}{A(1 - q)} \Rightarrow \frac{(1 - A)\lambda q}{A(1 - q)} > 0
\]

\[
(1 - A)\lambda 1 > 0 \Rightarrow 1 > A
\]

Which leads to the decision rule:

\[
x_i(q_i, 0) = \begin{cases} 
0 & \text{if } A \leq \frac{\lambda q}{\lambda q + (1 - q)} \\
1 - \frac{(1 - A)\lambda q}{A(1 - q)} & \text{if } \frac{\lambda q}{\lambda q + (1 - q)} < A < 1 \\
1 & \text{if } A = 1
\end{cases}
\]

(A.5)

Where $A$ is the solution to $\delta^2 (1 - \lambda) A^2 - (\delta + \delta^2) A + 1 = 0$

\[\square\]

A.1.7 Proof of proposition 1.7

Proposition 1.7. A duopolistic CRA will in period T-2, where the game ends in period T adopt the following rating strategy:

\[
x_1(q_1, q_2) = \begin{cases} 
0 & \text{if } A' \leq \Phi_1 \frac{\lambda q_1}{\lambda q_1 + (1 - q_1)} \\
1 - \frac{(1 - A')\lambda q_1}{A'(1 - q_1)} & \text{if } \Phi_1 \frac{\lambda q_1}{\lambda q_1 + (1 - q_1)} < A' < \Phi_1 \\
1 & \text{if } A' = \Phi_1
\end{cases}
\]

(A.6)

Where $A'$ is the solution to

\[
0 = 1 - \left\{ \delta \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) + \delta^2 \left( \lambda \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) \right) \right. \\
\left. + \left( \frac{1}{\Phi_2} B' - \min(A', B') \Phi_1 \max(\Phi_1, \Phi_2) \right) \left[ \lambda \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) + (1 - \lambda) \right) \right. \\
\left. (1 - q_2) \left( \frac{1}{\Phi_1} A' + (1 - \lambda) q_2 A'_{q_2=1} \right) + (1 - \max(A', B')) \min(\frac{1}{\Phi_1}, \frac{1}{\Phi_2}) \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) \right\}
\]

and $B' = \Phi_2(q_2, 1, q_1^N, 1) \frac{\mu(q_2) - c_H}{c_H - c_L}$

\[\text{Proof.}\] We know from proposition 1.5 that all opportunistic CRAs will play $x = 1$ in the last and second last period. We therefore know that the division of the market in period $T$ and $T-1$
will be determined from (3.6):
\[
\Phi|_{R_{i,j} > 0}(q_i, 1, q_j, 1) = \frac{1}{2} - \frac{\lambda}{2(1 - \lambda)p(q_i, 1)} + \frac{\lambda}{2(1 - \lambda)p(q_j, 1)} \\
= \frac{1}{2} - \frac{1 - (1 - \lambda)q_i}{2(1 - \lambda)} + \frac{1 - (1 - \lambda)q_j}{2(1 - \lambda)} \\
= \frac{1 + q_i - q_j}{2} \tag{A.7}
\]

Following the argument from the monopoly case, if a failure occurs the responsible CRA is eliminated from the game as its \( q \) falls to 0. Without any lose of generality it is assumed that CRA\(_1\) is approached for a rating in period \( T - 2 \). The gain from lying in period \( T - 2 \) is solely this periods fee:

\[
V(\text{lie}) = \phi
\]

The gain telling the truth in period \( T - 2 \) is:

\[
V(\text{honest}) = \left( \sum_{T=2}^{T-2} + \delta f_1^2(q_1^N, 1, q_2) \phi \right) + \left( \sum_{T=1}^{T-1} + \delta f_1^2(q_1^N, 1, q_2) \phi \right) + \left( \sum_{T=T-2}^{T-1} + \delta f_1^2(q_1^N, 1, q_2) \phi \right) + \left( \sum_{T=T-1}^{T-2} + \delta f_1^2(q_1^N, 1, q_2) \phi \right)
\]

where \( f_1^t(q_1, x_1, q_2, x_2) \) is the probability that CRA\(_i\) is approached for a rating in period \( t \in (1 = T - 2, 2 = T - 1, 3 = T) \). We know that if \( V(\text{lie}) > V(\text{honest}) \) the opportunistic CRA will always play \( x = 1 \) and vice versa \( x = 0 \). We therefore look at the case where \( V(\text{lie}) - V(\text{honest}) = 0 \).

To do so we define four variables:

\[
A = \frac{p(q_1^N) - c_H/I}{c_H/I - c_L/I} \\
A' = \Phi_1(q_1^N, 1, q_2, 1)A \\
B = \frac{p(q_2) - c_H/I}{c_H/I - c_L/I} \\
B' = \Phi_2(q_2, 1, q_1^N, 1)B
\]

From \( A' \) and \( B' \) it is possible to define the expression for \( V(\text{lie}) - V(\text{truth}) = 0 \) as:
\[ V(lie) - V(truth) = \phi - \phi \left\{ \delta \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) + \delta^2 \left( \lambda \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) \right) \right\} (A.8) \]

\[
\begin{align*}
\delta^2 \left( \lambda \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) \right) + (1 - \lambda)(1 - q_2) \frac{1}{\Phi_1} A' + (1 - \lambda)q_2 A'_{q_2=1} \\
+ (1 - \max(A', B') \min(\frac{1}{\Phi_1}, \frac{1}{\Phi_2}) \left( \frac{1}{\Phi_1} A' - \min(A', B') \Phi_2 \max(\Phi_1, \Phi_2) \right) \right) \} = 0 
\end{align*}
\]

We then proceed to show there exists an unique solution to the expression in terms of \( A' \).

To do so we solve for the two cases \( A' > B' \) and \( B' > A' \).

For \( A' < B' \)

Given the definitions for \( A' \) and \( B' \) we must have that \( q_2 > q_1^N \) which means that given (A.7) \( \Phi_1 < \Phi_2 \). Further we have that \( A', B' < 1 \) and that \( \Phi_1 + \Phi_2 = 1 \) which means that \( \Phi_1 < \frac{1}{2} \) and \( A' < \frac{1}{2} \). We then look at the case where \( V(truth) \) is greatest in period \( T - 2 \), given a bad project is drawn and \( CRA_1 \) is approached. We know that \( V(lie) = \phi \) if on the other hand the \( CRA \) chooses to tell the truth it will receive. \( V(truth) = 0 + \int_0^{T-1} \delta \phi + \int_0^{T} \delta^2 \phi \), where \( \bar{f} \) is the aggregated possibility for \( CRA_1 \) of receiving the business in period \( t \).

As proposed above the maximum attainable values of \( \Phi_1 \) and \( A' \) is: \( \Phi_1 = \frac{1}{2} - \epsilon, \epsilon \rightarrow 0 \) \( \Rightarrow A = 1 - \epsilon, \epsilon \rightarrow 0 \). Thus the highest possible value of \( V(truth) \) must be: \( V(truth) = 0 + (\frac{1}{2} - \epsilon)\delta \phi + (\frac{1}{2} - \epsilon)\delta^2 \phi \), given \( \delta \leq 1 \) we must have that \( V(lie) > V(truth) \). Therefore there is no solution to \( V(lie) - V(truth) = 0 \) for \( A' < B' \).

Which means that for \( A' < B' \) an opportunistic will always play \( x = 1 \). However \( q^N(x = 1) = 1 \) and \( \max(q_2) = 1 \) therefore it can never be the case that \( A' < B' \).

For \( A' > B' \)

\[
V(lie) - V(truth) = \\
1 - \delta \left( \frac{1}{\Phi_1} A' - \frac{1}{\Phi_2} B' \right) \Phi_2 + \delta^2 \left[ \lambda \left( \frac{1}{\Phi_1} A' - \frac{1}{\Phi_2} B' \right) \right] + \frac{1}{\Phi_2} \left( \frac{1}{\Phi_1} A' - \frac{1}{\Phi_2} B' \right) + (1 - \lambda)(1 - q_2) \frac{1}{\Phi_1} A' + (1 - \lambda)q_2 A'_{q_2=1} \\
+ (1 - \frac{1}{\Phi_1} A' \left( \frac{1}{\Phi_1} A' - \frac{1}{\Phi_2} B' \right) \right) \} 
\]

Similar to the proof for property 1.6 we define a second degree polynomial in terms of \( A' \) to
obtain a solution. We assume that $\delta = 1$.  

\[ A' = 2B' + \epsilon, \epsilon \geq 0 \]  

To show this we study the limiting case where the minus-sign applies. By adding $B'(2\frac{1}{\Phi'_1}(1-\lambda)) - B'(2\frac{1}{\Phi'_1}(1-\lambda))$ to the expression above we obtain:

\[ A' = B' + \frac{2 + B'(\frac{1}{\Phi'_1}(2(1-\lambda) + \lambda q_2) + (1-\lambda)q_2 \frac{\Phi_1 q_1}{2}]}{2(\frac{1}{\Phi'_1}(1-\lambda))} - \rho \]

where

\[ \rho = -B'^2 \frac{1}{\Phi'_1}(1-\lambda)^2 + 4B' \frac{1}{\Phi'_1}(1-\lambda) \left( 2\frac{1}{\Phi'_1} + B'(\frac{1}{\Phi'_1}(2(1-\lambda) + \lambda q_2) + (1-\lambda)q_2 \frac{\Phi_1 q_1}{2} ) \right) - 4 \frac{1}{\Phi'_1}(1-\lambda)(2B' + 1) \]

When $\rho > 0$ there always exists a solution to $A'$. The expression for $\rho$ is proportional to the second degree polynomial of $B'$ on the form:

\[ \Gamma(B') = B'^2 \left[ \frac{1}{\Phi'_1}(2(1-\lambda) + \lambda q_2) - (1-\lambda)(\frac{1}{\Phi'_1} - \frac{q_1 q_2}{2} ) \right] + B' \left[ 2\frac{1}{\Phi'_1} - 1 \right] - \frac{1}{c} \]

From this it follows that if $a > 0$ thus the expression, $\Gamma(B')$ must be convex.

\[ a > 0 \Rightarrow \frac{1}{\Phi'_1}(2(1-\lambda) + \lambda q_2) > (1-\lambda)(\frac{1}{\Phi'_1} - \frac{q_1 q_2}{2}) \Rightarrow 2 + \Phi'_1 q_1 q_2 + \frac{\lambda}{(1-\lambda)} q_2 > \frac{1}{\Phi_1} \]

Since $\frac{1}{\Phi'_1} \in [1, 2]$ we have that:

\[ \Phi'_1 q_1 q_2 + \frac{\lambda}{(1-\lambda)} q_2 > 0 \]

This is valid for $q_1, q_2 > 0$, i.e. duopoly case.

Since $B' = \Phi_2(q_2, 1, q_1^N, 1)B < A' \Rightarrow q_1^N > q_2$ we must have that $B' \in [0, \frac{1}{2}]$, $\Gamma(B' = 0) = -1$ and following the above $\Gamma(B')$ is convex, therefore when $\Gamma(B' = \frac{1}{2}) \leq 0$ then all values of $\Gamma$ in

\[ \text{By setting } \delta = 1 \text{ we find the case where the CRA will have the least incentive to lie.} \]
the interval $B' \in [0, \frac{1}{2}]$ must all be non-positive. For $B' = \frac{1}{2}$ we must have that $\Phi_2 = \frac{1}{2}$ and $B = 1$ which is only attainable when $q_1 = q_2 = 1$ thus we must have that:

$$
\Gamma(B' = \frac{1}{2}) = \frac{1}{4} (4(2(1 - \lambda) + \lambda) - (1 - \lambda)(8 - \frac{1}{2})) + \frac{1}{2} (2(1 - \lambda) - \frac{3}{4} - \frac{13}{2} (1 - \lambda) \leq 0
$$

This proofs that there always exists an solution for $A'$ as long as $A' > B'$.

We now turn to the second stage of the derivation, by solving for $A'$ with respect to $x_{1,T-2}$. We use the definitions for $A, A'$ and (2.2) and that the result from (A.4) still is valid for $A$.

$$
\frac{1}{\Phi_1} A' = \lambda q_1 N \frac{\lambda q_1 N}{\lambda q_1 N + 1 - q_1 N}
$$

$$
\frac{1}{\Phi_1} A'(\lambda q_1 + 1 - (1 - q_1) x_{1,T-2} q_1) = \lambda q_1
$$

$$
x_{1,T-2} = 1 - \frac{\lambda q_1 (1 - \frac{1}{\Phi_1} A')}{\frac{1}{\Phi_1} A'(1 - q_1)}
$$

$$
x_{1,T-2} = 1 - \frac{\lambda q_1 (1 - A)}{A (1 - q_1)}
$$

(A.9)

We then find the boundary conditions for A.9 given that $x_{T-2} \in [0,1]$.

$$
x_{T-2} = 1 \Rightarrow A = 1
$$

and

$$
x_{T-2} = 0 \Rightarrow A = \frac{\lambda q_1}{1 - (1 - \lambda) q_1}
$$

We thus get a decision rule of $x_{T-2}$ in terms of $A$ and $q_1$.

Since we ruled out the case where $A' < B'$ the equilibrium rating strategy for $CRA_1$ is:

$$
x_{1,T-2}(q_1, q_2) = \begin{cases} 
0 & \text{if } A \leq \frac{\lambda q_1}{\lambda q_1 (1 - q_1)} \\
1 - \frac{(1 - A) \lambda q_1}{A (1 - q_1)} & \text{if } \frac{\lambda q_1}{\lambda q_1 (1 - q_1)} < A < 1 \\
1 & \text{if } A = 1
\end{cases}
$$

(A.10)

where $A$ is the solution to (A.8). We see that $x_1$ is decreasing in $q_1$, i.e. as the CRA reputation increases the probability of being approached in both upcoming periods increase thus expected future profits increase. Remember lying entails a type revealing failure for sure, so $V(q_F)$ is constant so $q_N$ must fall to make $\phi$ equal to $V(q_N)$. Further, by numerical simulation we show that $x_1$ is increasing in $q_2$, see figure A.1. 19 As the reputation of the competing CRA increases

19We have verified through multiple parameter values, that this result is robust. Graphs can be e-mailed upon request.
Figure A.1: Strategy of a strategic doupol CRA for different values for competitor CRAs reputation, $q_2$

$\lambda, p_g, \delta, p_b = (1, 0.5, 1, 0.0)$

he is more likely to be approach in one of the following periods, thus lowering the expected future profits of the CRA approached in this period.
References


